INFLATIONARY EXPECTATIONS
AND THE
TERM STRUCTURE OF INTEREST RATES

BY

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1981

ACKNOWLEDGEMENTS

To my patient and confident adviser, Professor William A. Bomberger, I would like to express my deepest gratitude and sincerely say thank you very much for all that you have done for me.

I also wish to thank Professors Leonall C. Andersen, David A Denslow, H. Russell Fogler, and James T. McClave for their reading and comments on this manuscript.

I am also grateful to Michael Conlon for his invaluable computer programming assistance and to Barbara Long for her superb typing of the manuscript.

I wish to thank Patti Dunett of Action Graphics, Waco, Texas for her excellent graphics work.

Finally, I would like to express my sincere appreciation for the understanding and support of three special people: my wife, Barbara, my mother, and my father.

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March 1981

Chairman: William A. Bomberger
Major Department: Economics

Individuals find it to their advantage to gather information about the sources of inflation. In this study I assume they use this information by forming rational forecasts based on the structure of the process which precedes inflation. Individuals then are assumed to use the most important variables of that process in order to form more accurate forecasts of the future course of inflation.

The stochastic processes which generate those patterns of inflationary expectations for the different monthly horizons were then matched with interest rates of different monthly horizons. This was to test if the horizon of inflationary expectations matched the horizon of the same interest rate as the best explanatory variable for the

Fisher equation

$$r_t = \rho_t^e + \pi_t + \rho_t \pi_t^e \quad . \tag{1}$$

The r_t represents the nominal rate of interest at time period t, ρ_t represents the real rate of interest determined by the intersection of the supply and demand curves (roughly speaking the net savings and net investment curves) from the real section of the economy at time t, π_t^e represents the expected future price appreciation at time t, and $\rho_t \pi_t^e$ is the crossproduct which is of such small magnitude that it is dropped.

Using this matching approach the paper investigates the single idea that the expected real returns from holding debts to maturity of different terms are more similar than the nominal returns. Therefore, as one examines the structure of interest rates at a point in time, part or all of that structure is a reflection of the underlying structure of expected inflation.

This study shows, both in a main period, January 1959 to December 1978, and in an inflationary subperiod, January 1965 to December 1978, the horizon of inflationary expectations does not hold as the best explanatory variable for the nominal interest rate of the same horizon. Thus the underlying structure of expected inflation is not completely reflected in the term structure of interest rates.

CHAPTER I
INTRODUCTION

Perhaps as never before, today's financial economist faces tremendous uncertainty in our present economy. Each day seems to bring big surprises as interest rates soar and fall and inflation spirals upward to previously unheard of rates. Indeed, the uncertainty of the present age seems to be the only thing we can be certain of.

One of the more interesting questions in the theory of interest rates then becomes, "How does inflation affect interest rates?" More specifically, "How does expected inflation affect the nominal (market) rate of interest?" It is rightly noted that expected inflation is not the only argument in interest rate determination, but these types of questions are important to economic theory because it is through their assessment that we evaluate the effect of inflation on the real cost of capital and the real return to investors. These questions are quite broad in scope and currently involve much disagreement over the manner in which "expectations" of inflation are formed as well as the actual number of determinants to be used in their formulation.

The purpose of this study is to test a single idea: the expected real returns from holding debts to maturity of different terms are more similar than the nominal returns. Therefore, as one examines the structure of interest rates at a point in time, part or all of that structure is a reflection of the underlying structure of expected inflation.

An approach which would shed some light on these and related issues of expectations theories would be to answer the question, "Do different patterns which provide the best predictors of inflation over a given horizon also provide the best explanation of the movements of market interest rates of comparable duration?" This then will indicate the underlying expectations structure and its relationship to the nominal interest rate structure.

Because we are studying the effects of expected inflation on interest rates of different durations, we are in effect studying the effects of expected inflation on the term structure of interest rates. Alternatively stated, the study becomes a study of the effect that expectation of inflation has on the relationship among different maturities of debt.

Background

Interest Rates and Inflation. Irving Fisher (1930) was the first to formulate a systematic theory of the business cycle. This theory involved money stock changes,

commodity price changes, and a disparity between money interest rates and real interest rates induced by the commodity price changes which then caused the cycle to evolve.

It is from this work of Fisher, which drew upon the earlier work of Henry Thornton (1802), that we obtain the hypothesis that the nominal interest rate moves in the opposite direction to changes in the value of money (therefore, the same direction as price changes). This proposition is generally presented in some form similar to

$$r_t = \rho_t + \pi_t^e + \rho_t \pi_t^e \qquad (1)$$

where r_t represents the nominal rate of interest at time period t, ρ_t represents the real rate of interest determined by the intersection of the supply and demand curves (roughly speaking the net savings and net investment curves) from the real section of the economy at time t, π_t^e represents the expected future price appreciation at time t, and $\rho_t \pi_t^e$ is the crossproduct which is of such small magnitude that it is dropped.

The hypothesis is then generally tested in some form of a distributed lag equation as

$$r_t = \rho_t + \pi_t^e \quad . \qquad (2)$$

Then the heroic assumption is made under the economic theory of a long-run classical equilibrium that the real

rate is equal to some constant plus an error term, with zero mean and finite variance,

$$\rho_t = \alpha_o + \varepsilon_t \quad . \tag{3}$$

With statistical independence of the terms on the righthand side in equation (2), the test equation generally has become

$$r_t = \alpha_o + \pi_t^e + \varepsilon_t \quad . \tag{4}$$

where

$$\pi_t^e = \sum_{i=o}^{T} \alpha_i P_{t-i} \quad . \tag{5}$$

That is, expectations of inflation are dependent on some past series of inflation, and (Sargent 1973C) the weights relating that influence sum to one:

$$\sum_{i=o}^{T} \alpha_i = 1 \quad . \tag{6}$$

The Term Structure of Interest Rates

The theoretical relationship between interest rates of different maturities likewise was being developed during this and subsequent periods. This question of the relationship between interest rates and maturity is commonly called the yield curve. Lutz (1940) restated this

classical relationship of the yield curve in terms of five propositions from which we develop our modern theories.

He hypothesized that given three assumptions: that everyone knows the future short-term rate, there are no transactions cost, and there did not exist any constraints on shifting investments for either the borrower or the lender, that the relationship between the short-term and long-term rates:

1. could be conceived of as the long-term rate being an average of the future short-term rates,
2. that the long-term rate could never fluctuate as widely as the short-term rate,
3. that it is possible that the long-term rate may move temporarily opposite to the short rate,
4. that the current yield to redemption of a long-term bond will be above the current short-term rates up to the maturity date of the bond is above the current short-term rate (and vice-versa), and
5. that the return on an investment for a given time is the same no matter in what form the investment is made:

$$\frac{\text{nominal interest rate + capital gains (or -capital loss)}}{\text{purchasing price}}$$

The excitement then comes when the assumption of certainty about future short-term rates is dropped. Allowing for uncertainty then creates theories based on expected future short-term rates being related to long-term rates.

Implications of Both Questions

Until the recent article by Modigliani and Shiller (1973) both bodies of economic theory, i.e., the question of inflation and the interest rate and the question of the term structure of interest rates, were being developed in isolation. The Modigliani and Shiller article appears to be the first attempt to combine the two questions into one. Their article, as well as others, views only one stochastic process as generating expectations for the debt maturities studied.

If one believes the Fisher hypothesis to be true, the fact that we observe different expected future short-term rates and negative real rates, *ex post*, may be compatible with the notion of more than one stochastic process generating the expectation equation during periods of transition from one long-run equilibrium to another long-run equilibrium.

CHAPTER II
THEORETICAL MODEL

Macroeconomic Model

Sargent (1972b) has pointed out that if we wish to study the relationship between inflationary expectations and the nominal rate of interest it is most appropriate to do so in the context of a macroeconomic system.

Of course, what is rational, or consistent, for the individual to use in formulating his expectations is a function of what one places within the model. The following simplified IS-LM model is presented in order to talk about possible different stochastic processes generating inflationary expectations for different horizons.

Let the production sector be described by the following IS curve:

$$Y_t = Y_t^F + \alpha_o + \alpha_1 \ (r_t - \pi_t^e) + v_t \tag{7}$$

where $\alpha_o > 0$ and $\alpha_1 < 0$, Y_t is the log of real output, Y_t^F is the log of real full-employment output, and v_t is the disturbance term.

The monetary sector is represented as the following LM curve:

$$m_t = Y_t + \beta_o + \beta_1 r_t + \varepsilon_t \tag{8}$$

where $\beta_1 < 0$ and m_t is defined as the log of real money balances, i.e., log MS_t - log PL_t, where MS_t is the nominal money supply and PL_t is the price level and ε_t represents the disturbance term.

The model's price adjustment equation represents an intermediate position between the Keynesians and Monetarists. Friedman (1970) described the price adjustment equation as the crucial difference in the way they close IS-LM models. Keynesians regard the money wage rate as being exogenously determined and assume that firms set prices equal to or proportional to marginal costs. Assuming that short-run marginal cost are constant this means constant prices and thus implies that PL_t is exogenous and that we need to solve for Y_t and r_t. Monetarists, on the other hand, assume output, Y_t, equals the exogenously given full-employment level, Y_t^F, and treat r_t and PL_t as endogenous variables.

This model assumes that the percentage change in money wages can be represented by an expectational version of the Phillips curve,

$$w_t = a\ U_t + b_t\pi_{t-1}^e \tag{9}$$

where $a < 0$ and $b = 1$. In this equation, a defines the short-run trade-off between the unemployment rate and the rate of wage inflation, w, for a given ${}_t\pi_{t-1}^e$, and b

measures the extent to which inflationary expectations are reflected in current wage changes. The short-run capital stock is fixed, K_o: thus real output is a function of the other input factor, i.e., employment

$$Y_t = f(N_t) \tag{10}$$

where $f'(N_t) > 0$.

Assuming the unemployment rate is a linear function about the full-employment level, $\bar{N}$, then

$$U = \frac{\bar{N}_t - N_t}{\bar{N}_t} = k(Y_t - Y_t^F) \tag{11}$$

where $k < 0$.

With the assumptions that prices are a constant markup on unit labor costs, no technological change, and constant labor productivity, then we may define the unit of output to be such that

$$P_t = w_t \quad . \tag{12}$$

Substituting (12) and (11) into (9) and rearranging terms, the Phillips Curve becomes

$$Y_t = Y_t^F + \gamma(P_t - {}_t\pi_{t-1}^e) + \eta_t \tag{13}$$

where $\gamma > 0$ and η_t is the disturbance term. If in the long run one held expectations based on some constant

level of past inflation, then $(P_t - \pi^e_{t-1})$ would equal zero and

$$Y_t = Y^F_t + \eta_t \qquad (13a)$$

The three equations of the system

$$Y_t = Y_t + \alpha_o + \alpha_1 (r_t - \pi^e_t) + v_t \qquad (7),$$

$$m_t = Y_t + \beta_o + \beta_1 r_t + \varepsilon_t \qquad (8), \text{ and}$$

$$Y_t = Y^F_t + \gamma \; (P_t - {}_t\pi^e_{t-1}) + \eta_t \qquad (13),$$

have four unknowns if π^e_t is treated for the moment as exogenous: m_t, Y_t, r_t, and P_t. Thus the system is underdetermined. However, m_t and P_t are related by the identity

$$m_t = M_t - P_t + m_{t-1} \qquad (14)$$

where M_t represents the exponential growth rate for the money supply i.e., log MS_t - log MS_{t-1}, and P_t represents the exponential growth rate for inflation, i.e., log PL_t - PL_{t-1}. With these four equations then the system can be solved in the sense that the four endogenous variables may be expressed as linear functions of the predetermined variables and the disturbance terms. Let the exponential growth rate for capacity output, log Y^F_t - log Y^F_{t-1}, be defined as G_t. Continue to substitute for the lagged endogenous variables in the solution for P_t

until the solution may be written as an infinite sum of weighted past exogenous variables. The solution for P_t is

$$P_t = j_o + \sum_{i=o}^{\infty} k_i\, m_{t-i} + \sum_{i=o}^{\infty} l_i\, G_{t-i} + \sum_{i=o}^{\infty} n_i\, {}_{t-i}\pi^e_{t-1-i}$$

$$+ \sum_{i=o}^{\infty} q_i v_{t-i} + \sum_{i=o}^{\infty} w_i \varepsilon_{t-i} + \sum_{i=o}^{\infty} s_i \eta_{t-i} \qquad (15)$$

where j_o, k_i, l_i, n_i, q_i, w_i, and s_i are simply removed coefficients that are functions of α_o, β_o, α_1, β_1, and γ which can in principle be solved as well.

Up until now we have been treating π^e_t as a constant, more specifically, ${}_t\pi^e_{t-1}$. This is justified on the basis that the inflationary variable has been determined in the prior period. We now wish to extend our model to future periods. Thus with the longer-run extensions of the model it becomes necessary to treat π^e_t as an endogenous variable. Assume that the objective of the individual is to forecast this variable minimizing the sum of errors in the forecasting process, i.e., the individual is rational if he minimizes the expected value of mean square forecast error,

$$\min E[e^2_{t+i}] = E[P_{t+i} - {}_{t+i}\pi^e_t]^2 \quad i=1,2,\ \ldots \qquad . \qquad (16)$$

This implies that if the forecast of inflation is to be rational then

$$_{t+i}\pi_t^e = E(P_{t+i}) \quad i=1,2\ \ldots \qquad . \tag{17}$$

For example, from (15) we can derive the $E(P_{t+1})$ and thus the function for $_{t+1}\pi_t^e$,

$$_{t+1}\pi_t^e = j_o + \sum_{i=1}^{\infty} k_i m_{t+1-i} + \sum_{i=1}^{\infty} l_i G_{t+1-i} + \sum_{i=1}^{\infty} n_i \pi^e_{t+1-i} + \sum_{i=1}^{\infty} q_i v_{t+1-i} + \sum_{i=1}^{\infty} w_i \varepsilon_{t+1-i} + \sum_{i=1}^{\infty} s_i\ \eta_{t+1-i} + k_o\ E(m_{t+1}) + l_o\ E(G_{t+1}) + n_o E(\pi^e_{t+1}) + q_o\ E(v_{t+1}) + w_o\ E(\varepsilon_{t+1}) + s_o E(\eta_{t+1}). \tag{18}$$

To estimate equation (18) directly it is necessary to have observations on all variables, including the expected rates of inflation. Unfortunately, direct observations on expectations for the various horizons are not widely available. Therefore, a proxy variable needs to be substituted for expected inflation. In the literature, the typical solution to this problem is to assume that expectations are generated by a distributed lag on past values of that variable which are observable. This will be the procedure followed in this case. However, the number of lagged variables will be restricted by use of max χ^2 method for autoregressive schemes. The lag structure generating expectations will follow an

extrapolation hypothesis that expected inflation equals a weighted average of past rates of inflation together with past rates of growth of the two other variables indicated in equation (18). Then just what information becomes important in the formation of rational expectations depends upon the stochastic processes which generate the R.H.S. variables, the assumptions we make about the error terms. For example, suppose that after first differencing all of the time series on the R.H.S. of (18) we approximate "white noise." Then the best estimate for the future value of the expected variable would be its present value plus some error term. This then would have some weight attached to it in the overall formulation of expectations. Likewise, if the time series did not convert to "white noise," but instead showed some pattern which contained information on the future value of its series, then those observations would be included by their weight in the formation of expectations.

Of course, the disturbance terms in (18) will be proxied, as well as "full capacity" output. Expectations themselves are unmeasurable and will require some proxy. It is proposed that in terms of equation (18) expectations may be represented as a function of the past series of monthly inflation rates, Consumer Price Index, monthly growth rate of the money supply, MIA, and the growth rate of monthly personal income, i.e.,

$$ {}_{t+1}\pi^e_t = f(P_t, P_{t-1}, P_{t-2}, \ldots ; M_t, M_{t-1}, M_{t-2}, \ldots ; $$

$$ Y_t, Y_{t-1}, Y_{t-2}, \ldots ; \dot{r}_t, \dot{r}_{t-1}, \dot{r}_{t-2}, \ldots). \qquad (19) $$

where the interest rate series serves as a proxy for $E(M_{t+1})$.

Let the exponential growth rate for expected inflation one period (one month) forward, log PL_{t+1} - log PL_t, be defined as ${}_{t+1}\pi^e_t$. Likewise, let the exponential growth rate for expected inflation two periods (two months) forward, log PL_{t+2} - log PL_t, be defined as ${}_{t+2}\pi^e_t$. Continue this definitional pattern for expected inflation forward for 60 periods. Thus the variable ${}_{t+60}\pi^e_t$ represents the 5 year expected inflation rate as of period t.

Other Variables

The theoretical model described above is not too interesting if the only conclusion is a mathematical manipulation, since virtually any result could be obtained depending on what is included in the model. What makes the "other variables" interesting is that they carry an implication for different stochastic processes over time. The economic interpretation of different processes for the short-term with several variables versus the long-term with only one variable has a very interesting explanation in terms of information-processing by the market place.

It is suggested that additional variables are necessary in the short-term equations because information sources (data) on which investors form short-term inflationary expectations are embedded with a high degree of unreliability and possibly conflict. Short-term forecasts of inflation are influenced by credit market conditions, while medium and long-term forecasts are not, and investors behave as though changes in fundamental economic relationships in the short-term are highly unpredictable and use several sources (different variables) of information to reduce their uncertainty. Also, within a shorter time period expectational effects occasioned by excluded variables, e.g., changes in the money stock, act to reduce (increase) the real rate of interest temporarily. That is, money is nonneutral in the short-term. This was tested in the above mentioned model by adding additional variables, e.g., money growth, etc., in the interest rate determination equation as the variable Z_t, i.e.,

$$r_t = \alpha_o + \beta_{1t+n}\pi_t^e + \beta_i Z_t + \varepsilon_t \quad i=2,3,4 \quad . \tag{20}$$

This new formulation for short-term expectations does not imply that the markets are not efficient. Rather, the markets are efficient, i.e., all the currently available information is in the market place, but it implies that our method of extracting the current information

about expected prices is inefficient if we restrict ourselves to only one time series. Thus, the information of how that series to be predicted will tend to move in the future is a function not only of its own past, but also of the indirect effects of other series, which act on the future course of the price series itself.

Roughly speaking this implies that a chnge in current personal income carries some information about the future disturbance term. If income is rising toward full employment income, it is valid to assume that this imparts expectations about prices rising in the future. The individual may even be said to expect prices to start rising prior to Y_t^F, rather than have some clean break assumption for price increase or decrease for $Y_t \neq Y_t^F$. Rapid changes in price levels may signal fear of escalating prices in the immediate future.

Likewise, current money supply changes may signal the path of future prices over the next few months. This information may or may not be reflected in the past price series. Therefore, changes in the money supply were included in the equation for the formation of expectations.

Finally, the last theoretical variable included in the category "other variables," i.e., variables other than the past series of prices, is the change in the

short-term interest rate, r_t. This is included on the grounds that it affects expectations of individuals regarding the future monetary policy of the government.

This analysis provides a useful approach to the two bodies of literature which analyze the role of inflationary expectations. The first of these deals with the question of the determinates in the formulation of inflationary expectations. The second of these deals with the question of testing Fisher's hypothesis that r_t fully adjusts to inflationary expectations, thus the sum of the coefficients of lagged prices equals one. The majority of the empirical studies yield estimates well below unity and these are taken to reject the hypothesis (Yohe and Karnosky 1969 and Gibson 1970). It is possible that the reason the coefficients sum to less than unity is because the specification of the formulation of expectations in these studies is incorrect for the short-term interest rates.

All of these "other variables" would have a positive effect on expectations of changes in prices at least in the immediate term. Thus they would have an indirect positive effect on the short-term interest rate.

With respect to the long-term interest rates, all information about inflationary expectations would be contained in a single variable time series, the price series. That is, as the time horizon lengthens then the basis for

the formulation of expectations about inflation should be reduced from an informational set of four series to one series because all relevant information has passed from the other series to the price series.

These different stochastic processes then answer questions about the determination of the term structure of interest rates. They imply that as the informational set is unrestricted that the empirical conflict in the literature will be reduced, as R. Craine and J. L. Pierce (1978) claim. It almost seems counterintuitive to imply that the agent is more certain about a five-year forecast of inflation than about a two-year or three-year forecast. But perhaps the individual knows the eventual outcome of fundamental economic relationships, though highly uncertain of the short-term timing for those events.

CHAPTER III
TESTING AND RESULTS

The hypothesis that the market rate of interest on debt of different durations can best be explained by inflationary expectations formed over different horizons that correspond to those debt horizons is a joint hypothesis. The first part, a test for the formulation of expectations, is described below as the preliminary step. The preliminary step is necessary since inflationary expectations are constructed by statistical modeling. The second part, a test matching the different maturities of interest rates and inflationary expectations, is designed to answer the question, "How much of the structure of interest rates is caused by the structure of expected inflation?" The second part is designated below as the main test.

Finally, a control is conducted for the main test results by repeating the main test again for a subperiod which has a different inflation rate. The control, designated below as the subperiod test, allows comparison and contrast of two different inflation periods.

Preliminary Step to the Test

The preliminary step to the test is based on the additional explanatory power yield by the additional variables in the expected inflation equation. The traditional linear autoregressive structure:

$$P_t = \alpha + \beta_1 P_{t-1} + \beta_2 P_{t-2} + \ldots + \varepsilon_t \tag{21}$$

is derived from a generalized polynomial function with power terms for the various P's including time and its powers as variables:

$$P_t = \sum_{i=o}^{\infty} \alpha_i t^i + \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \beta_j P_{t-j}^k + \varepsilon_t \tag{22}$$

under the assumption that the linear specification is correct and will thus lead to an unbiased and efficient forecast. The best prediction made at the end of the current month for the next month t+1 then becomes

$$_{t+1}\pi_t^e = a + b_1 P_t + b_2 P_{t-1} + \ldots + 0 \tag{23}$$

where the forecast error is

$$\varepsilon_{t+1} = (P_{t+1} - {}_{t+1}\pi_t^e) \quad . \tag{24}$$

The purpose of including additional autonomous variables is to reduce the size of the error term. For each duration of actual inflation there exist several possible autoregressive schemes. The best two, one a single

series model and the other a multiple series model, are pitted against one another to select the better model in terms of the minimum size error term.

Thus, consider the equations for expected inflation one month hence tested for fit with the following two autoregressive schemes:

$$\text{(A)} \quad {}_{t+1}\pi_t^e = \phi(L)P_{t-i} \quad i=0,\ 1,\ \ldots \tag{25}$$

versus

$$\text{(B)} \quad {}_{t+1}\pi_t^e = \phi(L)P_{t-i} + \theta(L)M_{t-i} + \Psi(L)Y_{t-i} +$$

$$\Omega(L)\ \dot{r}_{t-i} \quad i=0,\ 1,\ \ldots \tag{26}$$

where $\phi(L)$, $\theta(L)$, $\Psi(L)$, and $\Omega(L)$ represent the coefficients for each lagged polynomial, and the other variables are as described before.

The preliminary step should be considered as a set of screening steps which, after Box-Jenkins (1976) identification for differencing requirements, divides the set of possible autoregressive orders into those of an efficient set and those of an inefficient set for both the single series case and the multiple series case. This division into subsets is accomplished for the single series case by performing the max χ^2 operation on the differenced price series. The optimal lag list is then used to construct the single series model. The multiple

series model requires that P_t be regressed onto M_t, Y_t, and $\dot{r}_t$

$$P_t = \beta_1 + \beta_2 M_t + \beta_3 Y_t + \dot{r}_t + \varepsilon_t \tag{27}$$

by OLS regression and the residuals from the regression, ε_t, be operated on by the max χ^2 procedure. Selection is made then from the efficient set of that alternative ARI order which, according to the max χ^2 method, minimizes the residual variance for each case, i.e., the single series case and the multiple series case. Finally, one selects between alternatives, the single series equation and the multiple series equation, the better method of describing inflationary expectations for each duration.

For example, the choice of the equation to represent inflationary expectations one month hence, n=1, was between the single series model:

$${}_{t+1}\pi_t^e = \underset{(.06)}{.31}\,P_t + \underset{(.06)}{.36}\,P_{t-1} + \underset{(.06)}{.28}\,P_{t-5} \tag{28}$$

with summary statistics:

$\bar{R}^2$ .78	DW 2.15
MSE 4.8E-6	Prob > F .0001

and the multiple series model:

$$
\begin{aligned}
{}_{t+1}\pi^e_t = {} & \underset{(.06)}{.21}\, P_t + \underset{(.06)}{.30}\, P_{t-1} + \underset{(.06)}{.24}\, P_{t-5} + \underset{(.05)}{.10}\, M_t + \\
& \underset{(.05)}{.06}\, M_{t-1} + \underset{(.05)}{.01}\, M_{t-4} - \underset{(.03)}{.00}\, Y_t - \underset{(.03)}{.01}\, Y_{t-1} + \\
& \underset{(.00)}{.06}\, Y_{t-4} + \underset{(.00)}{.00}\, \dot{r}_t + \underset{(.00)}{.00}\, \dot{r}_{t-1} + \underset{(.00)}{.00}\, \dot{r}_{t-4}
\end{aligned}
\tag{29}
$$

with summary statistics:

$\bar{R}^2$ .80	DW 2.15
MSE 4.6E-6	Prob > F .0001 .

The values in parenthesis represent the standard errors. The symbol ${}_{t+1}\pi^e_t$ represents the expected inflation rate, log $\hat{PL}_{t+1} - PL_t$. Likewise, P_t represents the actual difference in the past log levels of the monthly Consumer Price Index, log PL_t - log PL_{t-1}, P_{t-1} represents log PL_{t-1} - log PL_{t-2}, and P_{t-5} represents log $PL_{t-5} - PL_{t-6}$. The actual difference in the past log levels of the monthly money supply (M1, currency plus demand deposits) is represented by M_{t-1}, where i=0, 1, 2, ... In equation 29, M_t represents log MS_t - log MS_{t-1}, M_{t-1} represents log MS_{t-1} - log MS_{t-2}, and M_{t-4} represents log MS_{t-4} - log MS_{t-5}. The symbol Y_{t-i}, where i=0, 1, 2, ... , represents changes in the past levels of monthly Personal Income. For example, Y_t represents log PI_t - log PI_{t-1}, Y_{t-1} represents log

PI_{t-1} - log PI_{t-2}, and Y_{t-4} represents log PI_{t-4} - log PI_{t-5}. Finally $\dot{r}_{t-i}$, where i=0, 1, 2, ... , represents the difference in the end of the month annualized bond-equivalent yield of one month seasoned U.S. Government Treasury Bills. In this case $\dot{r}_t$ represents $r_t - r_{t-1}$, $\dot{r}_{t-1}$ represents $r_{t-1} - r_{t-2}$, and $\dot{r}_{t-4}$ represents $r_{t-4} - r_{t-5}$.[1]

In the above manner then one is able to select the multiple series model as the better fit for expected inflation one month forward, ${}_{t+1}\pi^e_t$, during the period January 1959 to December 1978.

To extend this process forward for t+n periods, we make use of the general equation for single series forecasting:

$$
{}_{t+n}\pi^e_t = \phi_1({}_{t+n-1}\pi^e_t) + \phi_2({}_{t+n-2}\pi^e_t) + \dots + \phi_{n-1}({}_{t-n+1}\pi^e_t) + \phi_n P_{t-n} + \phi_{n+1} P_{t-n-1} + \dots \tag{30}
$$

which gives an optimal forecast in the sense of the minimum extrapolation error (Mincer 1969).

For example where n=2, i.e., ${}_{t+2}\pi^e_t$, substitute the as yet unknown magnitude into the autoregression by its extrapolated value. Thus the traditional formulation:

$$
{}_{t+1}\pi^e_t = \alpha + \phi_1 P_t + \phi_2 P_{t-1} + \dots \tag{31}
$$

is substituted into

$$ {}_{t+2}\pi_t^e = \alpha + \phi_1 \hat{P}_{t+1} + \phi_2 P_t + \phi_3 P_{t-1} + \ldots \qquad (32) $$

for $\hat{P}_{t+1}$, giving

$$ {}_{t+2}\pi_t^e = \alpha + \phi_1 ({}_{t+1}\pi_t^e + \varepsilon_{t+1}) + \phi_2 P_t + \phi_3 P_{t-1} + \ldots + \varepsilon_{t+2} \quad . \qquad (33) $$

Thus the mean squared error of extrapolation in predicting ${}_{t+2}\pi_t^e$, the two month expected inflation at time period t, for the single series forecast is the variance of $(\phi_1 \varepsilon_{t+1} + \varepsilon_{t+2})$.

Likewise, given that all the R.H.S. series of the multivariate model have been whitened with constant variance, the alternative formulation for the multivariate scheme is

$$ \begin{aligned} {}_{t+2}\pi_t^e = \alpha &+ \phi_1 ({}_{t+1}\pi_t^e + \varepsilon_{t+1}^P) + \phi_2 P_t + \phi_3 P_{t-1} + \ldots + \\ &\theta_1 ({}_{t+1}M_t^e + \varepsilon_{t+1}^m) + \phi_2 M_t + \phi_3 M_{t-1} + \ldots + \\ &\Psi_1 ({}_{t+1}Y_t^e + \varepsilon_{t+1}^Y) + \Psi_2 Y_t + \Psi_3 Y_{t-1} + \ldots + \\ &\Omega_1 ({}_{t+1}\dot{r}_t^e + \varepsilon_{t+1}^{\dot{r}}) + \Omega_2 \dot{r}_t + \Omega_3 \dot{r}_{t-1} + \ldots + \varepsilon_{t+2} \quad . \end{aligned} \qquad (34) $$

Therefore, the mean squared error in predicting ${}_{t+2}\pi_t^e$ with the multivariate model is the variance of $(\phi_1 \varepsilon_{t+1}^P + \theta_1 \varepsilon_{t+1}^m + \Psi_1 \varepsilon_{t+1}^Y + \Omega_1 \varepsilon_{t+1}^{\dot{r}} + \varepsilon_{t+2})$.

Renaming the coefficients to eliminate rewriting the constants and bracketed terms, the choice is between

$$ {}_{t+2}\pi_t^e = \phi^*(L)P_{t-i} \quad i=0,\ 1,\ \ldots \tag{35}$$

and

$$ {}_{t+2}\pi_t^e = \phi^*(L)P_{t-i} + \theta^*(L)M_{t-i} + \Psi^*(L)Y_{t-i} + \Omega^*(L)\dot{r}_{t-i} \quad i=0,\ 1,\ \ldots \tag{36}$$

for the two period forecast.

For the horizon n=2, the best single series model was:

$$ {}_{t+2}\pi_t^e = \underset{(.11)}{.60}\, P_t + \underset{(.10)}{.57}\, P_{t-1} + \underset{(.10)}{.32}\, P_{t-2} + \underset{(.10)}{.42}\, P_{t-5} \tag{37}$$

with summary statistics:

$\bar{R}^2$	.85	DW	1.14
MSE	1.2E-5	Prob > F	.0001

The best of the multiple series models, and the better of the single series versus multiple series models, was:

$$ {}_{t+2}\pi_t^e = \underset{(.10)}{.44}\, P_t + \underset{(.10)}{.45}\, P_{t-1} + \underset{(.10)}{.22}\, P_{t-2} + \underset{(.10)}{.35}\, P_{t-5} + $$

$$ \underset{(.07)}{.17}\, M_t + \underset{(.08)}{.11}\, M_{t-1} + \underset{(.07)}{.07}\, M_{t-2} - \underset{(.07)}{.00}\, M_{t-4} + $$

$$ \underset{(.04)}{.01}\, Y_t - \underset{(.05)}{.00}\, Y_{t-1} - \underset{(.05)}{.00}\, Y_{t-2} + \underset{(.05)}{.12}\, Y_{t-4} + $$

$$\underset{(.00)}{.00}\,\dot{r}_t - \underset{(.00)}{.00}\,\dot{r}_{t-1} - \underset{(.00)}{.00}\,\dot{r}_{t-2} + \underset{(.0004)}{.0008}\,\dot{r}_{t-4} \qquad (38)$$

with summary statistics:

$\bar{R}^2$ .88	DW 1.12
MSE 1.1E-5	Prob > F .0001

In summary, the technique used to forecast the forward one period rate of inflation for periods where $n > 1$ is that after the usual Box-Jenkins identification for stationarity and the initial preliminary max χ^2 screen for the optimal lag list, set up a two phase least squares procedure. The lag list for the future values of the variables for time periods less than n from the first OLS regression are then used as R.H.S. variables in the estimation procedure of the second OLS regression. In this manner, the estimation procedure steps the forecast forward one step at a time giving the one period forward rate of expected inflation for that duration and assures approximately the same variance for each forward rate.

Thus by operating on the autoregressive schemes forward, applying Wold's "chain principle of forecasting," replacing stochastic elements on the R.H.S. with their prior lag last, expectations of inflation were derived for different durations. Table 1 then shows the best autoregressive scheme using the max χ^2 approach, and has several interesting points for later discussion.

Table 1
The Term Structure of Inflationary Expectations, $_{t+n}\pi^e_t$
1/59 to 12/1978

Monthly Horizon, t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+1}\pi^e_t$	=	$\underset{(.064)}{.206}\, P_t + \underset{(.062)}{.301}\, P_{t-1} + \underset{(.059)}{.243}\, P_{t-5} + \underset{(.045)}{.100}\, M_t + \underset{(.047)}{.060}\, M_{t-1} +$				
		$\underset{(.046)}{.007}\, M_{t-4} - \underset{(.029)}{.001}\, Y_t - \underset{(.030)}{.012}\, Y_{t-1} + \underset{(.029)}{.057}\, Y_{t-4} + \underset{(.0003)}{.0004}\, \dot{r}_t +$				
		$\underset{(.0003)}{.0001}\, \dot{r}_{t-1} + \underset{(.0003)}{.0003}\, \dot{r}_{t-4}$	.80	4.6E-6	.0001	2.15
$_{t+2}\pi^e_t$	=	$\underset{(1.04)}{.444}\, P_t + \underset{(.099)}{.450}\, P_{t-1} + \underset{(.102)}{.220}\, P_{t-2} + \underset{(.095)}{.354}\, P_{t-5} + \underset{(.075)}{.171}\, M_t +$				
		$\underset{(.075)}{.108}\, M_{t-1} + \underset{(.073)}{.067}\, M_{t-2} - \underset{(.073)}{.001}\, M_{t-4} + \underset{(.045)}{.008}\, Y_t - \underset{(.046)}{.002}\, Y_{t-1} -$				
		$\underset{(.047)}{.005}\, Y_{t-2} + \underset{(.045)}{.118}\, Y_{t-4} + \underset{(.0005)}{.0003}\, \dot{r}_t - \underset{(.0005)}{.0003}\, \dot{r}_{t-1} -$				
		$\underset{(.0005)}{.0006}\, \dot{r}_{t-2} + \underset{(.0004)}{.0008}\, \dot{r}_{t-4}$	.88	1.1E-5	.0001	1.12
$_{t+3}\pi^e_t$	=	$\underset{(.144)}{.597}\, P_t + \underset{(.141)}{.559}\, P_{t-1} + \underset{(.144)}{.359}\, P_{t-2} + \underset{(.141)}{.331}\, P_{t-5} + \underset{(.145)}{.367}\, P_{t-7} +$				
		$\underset{(.102)}{.284}\, M_t + \underset{(.106)}{.138}\, M_{t-1} + \underset{(.102)}{.153}\, M_{t-2} + \underset{(.102)}{.038}\, M_{t-4} + \underset{(.107)}{.204}\, M_{t-7} +$				
		$\underset{(.063)}{.019}\, Y_t - \underset{(.065)}{.014}\, Y_{t-1} + \underset{(.066)}{.015}\, Y_{t-2} + \underset{(.064)}{.105}\, Y_{t-4} - \underset{(.064)}{.081}\, Y_{t-7} +$				
		$\underset{(.0007)}{.0002}\, \dot{r}_t - \underset{(.0007)}{.0004}\, \dot{r}_{t-1} + \underset{(.0007)}{.0001}\, \dot{r}_{t-2} + \underset{(.0006)}{.0001}\, \dot{r}_{t-4} +$				
		$\underset{(.0006)}{.0009}\, \dot{r}_{t-7}$	.90	2.0E-5	.0001	.98

Table 1 Cont'd

Monthly Horizon, t+n	Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+4}\pi^e_t$ =	$\underset{(.179)}{.678} P_t + \underset{(.175)}{.728} P_{t-1} + \underset{(.179)}{.508} P_{t-2} + \underset{(.178)}{.536} P_{t-5} + \underset{(.181)}{.426} P_{t-7} +$				
	$\underset{(.120)}{.365} M_t + \underset{(.132)}{.144} M_{t-1} + \underset{(.128)}{.106} M_{t-2} + \underset{(.128)}{.041} M_{t-4} + \underset{(.134)}{.228} M_{t-7} +$				
	$\underset{(.079)}{.033} Y_t + \underset{(.081)}{.026} Y_{t-1} + \underset{(.082)}{.066} Y_{t-2} + \underset{(.079)}{.060} Y_{t-4} - \underset{(.080)}{.058} Y_{t-7} +$				
	$\underset{(.0009)}{.0002} \dot{r}_t - \underset{(.0008)}{.0003} \dot{r}_{t-1} + \underset{(.0008)}{.0004} \dot{r}_{t-2} + \underset{(.0007)}{.002} \dot{r}_{t-4} +$				
	$\underset{(.0008)}{.001} \dot{r}_{t-7}$	.91	3.1E-5	.0001	.81
$_{t+5}\pi^e_t$ =	$\underset{(.025)}{.753} P_t + \underset{(.202)}{.837} P_{t-1} + \underset{(.212)}{.469} P_{t-2} + \underset{(.203)}{.522} P_{t-4} + \underset{(.211)}{.597} P_{t-5} +$				
	$\underset{(.208)}{.563} P_{t-7} + \underset{(.148)}{.466} M_t + \underset{(.151)}{.200} M_{t-1} + \underset{(.149)}{.238} M_{t-2} + \underset{(.157)}{.200} M_{t-4} +$				
	$\underset{(.151)}{.049} M_{t-5} + \underset{(.158)}{.091} M_{t-7} + \underset{(.093}{.074} Y_t + \underset{(.093)}{.075} Y_{t-1} + \underset{(.094)}{.073} Y_{t-2} -$				
	$\underset{(.091)}{.004} Y_{t-4} - \underset{(.096)}{.079} Y_{t-5} - \underset{(.092)}{.081} Y_{t-7} + \underset{(.001)}{.0006} \dot{r}_t +$				
	$\underset{(.001\ )}{.0007} \dot{r}_{t-1} + \underset{(.001)}{.0009} \dot{r}_{t-2} + \underset{(.0009)}{.003} \dot{r}_{t-4} + \underset{(.0009)}{.003} \dot{r}_{t-5} +$				
	$\underset{(.0009)}{.001} \dot{r}_{t-7}$	.93	4.0E-5	.0001	.74

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+6}\pi^e_t$	=	$\underset{(.235)}{.939} P_t + \underset{(.234)}{.967} P_{t-1} + \underset{(.245)}{.483} P_{t-2} + \underset{(.234)}{.657} P_{t-4} + \underset{(.243)}{.661} P_{t-5} +$				
		$\underset{(.241)}{.643} P_{t-7} + \underset{(.170)}{.573} M_t + \underset{(.174)}{.208} M_{t-1} + \underset{(.171)}{.351} M_{t-2} + \underset{(.181)}{.197} M_{t-4} +$				
		$\underset{(.175)}{.035} M_{t-5} + \underset{(.182)}{.181} M_{t-7} + \underset{(.107)}{.125} Y_t + \underset{(.106)}{.052} Y_{t-1} - \underset{(.108)}{.005} Y_{t-2} -$				
		$\underset{(.105)}{.0002} Y_{t-4} - \underset{(.111)}{.049} Y_{t-5} + \underset{(.106)}{.008} Y_{t-7} + \underset{(.001)}{.001} \dot{r}_t + \underset{(.001)}{.0004} \dot{r}_{t-1} +$				
		$\underset{(.001)}{.001} \dot{r}_{t-2} + \underset{(.001)}{.004} \dot{r}_{t-4} + \underset{(.001)}{.004} \dot{i}_{t-5} + \underset{(.001)}{.002} \dot{r}_{t-7}$	.93	5.3E-5	.0001	.72
$_{t+7}\pi^e_t$	=	$\underset{(.261)}{1.020} P_t + \underset{(.268)}{.954} P_{t-1} + \underset{(.272)}{.616} P_{t-2} + \underset{(.267)}{.608} P_{t-3} + \underset{(.259)}{.634} P_{t-4} +$				
		$\underset{(.276)}{.749} P_{t-5} + \underset{(.266)}{.685} P_{t-7} + \underset{(.196)}{.725} M_t + \underset{(.196)}{.484} M_{t-1} + \underset{(.203)}{.637} M_{t-2} +$				
		$\underset{(.202)}{.112} M_{t-3} + \underset{(.206)}{.038} M_{t-4} - \underset{(.202)}{.137} M_{t-5} + \underset{(.200)}{.151} M_{t-7} + \underset{(.119)}{.117} Y_t -$				
		$\underset{(.120)}{.035} Y_{t-1} - \underset{(.120)}{.076} Y_{t-2} - \underset{(.120)}{.033} Y_{t-3} + \underset{(.117)}{.026} Y_{t-4} -$				
		$\underset{(.122)}{.074} Y_{t-5} - \underset{(.117)}{.021} Y_{t-7} + \underset{(.001)}{.001} \dot{r}_t + \underset{(.001)}{.001} \dot{r}_{t-1} + \underset{(.001)}{.003} \dot{r}_{t-2} +$				
		$\underset{(.001)}{.005} \dot{r}_{t-3} + \underset{(.001)}{.006} \dot{r}_{t-4} + \underset{(.001)}{.005} \dot{r}_{t-5} + \underset{(.001)}{.002} \dot{r}_{t-7}$	.94	6.3E-5	.0001	.67

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+8}\pi^e_t$	=	$\underset{(.286)}{.701} P_t + \underset{(.286)}{.926} P_{t-1} + \underset{(.287)}{.858} P_{t-2} + \underset{(.284)}{.752} P_{t-3} + \underset{(.283)}{.650} P_{t-4} +$				
		$\underset{(.292)}{.681} P_{t-5} + \underset{(.285)}{.687} P_{t-6} + \underset{(.286)}{.773} P_{t-7} + \underset{(.210)}{.856} M_t + \underset{(.213)}{.840} M_{t-1} +$				
		$\underset{(.218)}{.725} M_{t-2} + \underset{(.221)}{.262} M_{t-3} + \underset{(.221)}{.197} M_{t-4} + \underset{(.218)}{.084} M_{t-5} -$				
		$\underset{(.221)}{.202} M_{t-6} - \underset{(.223)}{.128} M_{t-7} + \underset{(.127)}{.073} Y_t - \underset{(.127)}{.077} Y_{t-1} - \underset{(.126)}{.147} Y_{t-2} -$				
		$\underset{(.127)}{.053} Y_{t-3} - \underset{(.128)}{.051} Y_{t-4} - \underset{(.130)}{.047} Y_{t-5} - \underset{(.124)}{.065} Y_{t-6} +$				
		$\underset{(.125)}{.031} Y_{t-7} + \underset{(.001)}{.002} \dot{r}_t + \underset{(.001)}{.003} \dot{r}_{t-1} + \underset{(.001)}{.004} \dot{r}_{t-2} + \underset{(.001)}{.005} \dot{r}_{t-3} +$				
		$\underset{(.001)}{.007} \dot{r}_{t-4} + \underset{(.001)}{.008} \dot{r}_{t-5} + \underset{(.001)}{.006} \dot{r}_{t-6} + \underset{(.001)}{.004} \dot{r}_{t-7}$	.95	6.9E-5	.0001	.44
$_{t+9}\pi^e_t$	=	$\underset{(.313)}{.796} P_t + \underset{(.312)}{.996} P_{t-1} + \underset{(.312)}{.883} P_{t-2} + \underset{(.309)}{.925} P_{t-3} + \underset{(.309)}{.664} P_{t-4} +$				
		$\underset{(.318)}{.674} P_{t-5} + \underset{(.311)}{.849} P_{t-6} + \underset{(.312)}{.857} P_{t-7} + \underset{(.234)}{.975} M_t + \underset{(.234)}{.938} M_{t-1} +$				
		$\underset{(.239)}{.803} M_{t-2} + \underset{(.241)}{.221} M_{t-3} + \underset{(.241)}{.325} M_{t-4} + \underset{(.239)}{.073} M_{t-5} -$				
		$\underset{(.241)}{.210} M_{t-6} + \underset{(.244)}{.226} M_{t-7} + \underset{(.138)}{.052} Y_t - \underset{(.139)}{.098} Y_{t-1} - \underset{(.138)}{.116} Y_{t-2} -$				
		$\underset{(.139)}{.060} Y_{t-3} + \underset{(.139)}{.032} Y_{t-4} - \underset{(.141)}{.091} Y_{t-5} - \underset{(.136)}{.019} Y_{t-6} + \underset{(.136)}{.070} Y_{t-7} +$				
		$\underset{(.001)}{.003} \dot{r}_t + \underset{(.001)}{.004} \dot{r}_{t-1} + \underset{(.002)}{.005} \dot{r}_{t-2} + \underset{(.001)}{.006} \dot{r}_{t-3} + \underset{(.002)}{.008} \dot{r}_{t-4} +$				
		$\underset{(.002)}{.009} \dot{r}_{t-5} + \underset{(.002)}{.007} \dot{r}_{t-6} + \underset{(.002)}{.004} \dot{r}_{t-7}$	.95	8.2E-5	.0001	.39

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+10}\pi^e_t$	=	.826 P_t (.343) + 1.014 P_{t-1} (.341) + 1.074 P_{t-2} (.341) + .975 P_{t-3} (.330) + .626 P_{t-4} (.338) +				
		.878 P_{t-5} (.349) + .971 P_{t-6} (.341) + .787 P_{t-7} (.342) + 1.141 M_t (.257) +				
		1.048 M_{t-1} (.256) + .748 M_{t-2} (.263) + .351 M_{t-3} (.264) + .336 M_{t-4} (.264) +				
		.070 M_{t-5} (.261) − .353 M_{t-6} (.264) − .153 M_{t-7} (.267) + .029 Y_t (.151) − .066 Y_{t-1} (.152) −				
		.117 Y_{t-2} (.151) + .024 Y_{t-3} (.152) − .013 Y_{t-4} (.52) − .051 Y_{t-5} (.155) +				
		.019 Y_{t-6} (.149) + .058 Y_{t-7} (.149) + .004 $\dot{r}_t$ (.002) + .005 $\dot{r}_{t-1}$ (.002) + .005 $\dot{r}_{t-2}$ (.002) +				
		.007 $\dot{r}_{t-3}$ (.002) + .009 $\dot{r}_{t-4}$ (.002) + .010 $\dot{r}_{t-5}$ (.002) + .007 $\dot{r}_{t-6}$ (.002) + .004 $\dot{r}_{t-7}$ (.001)	.96	9.8E-5	.0001	.38
$_{t+11}\pi^e_t$	=	.838 P_t (.376) + 1.193 P_{t-1} (.374) + 1.162 P_{t-2} (.374) + .996 P_{t-3} (.370) + .780 P_{t-4} (.371) +				
		.996 P_{t-5} (.383) + .907 P_{t-6} (.373) + .781 P_{t-7} (.374) + 1.265 M_t (.282) + 1.045 M_{t-1} (.282) +				
		.935 M_{t-2} (.288) + .362 M_{t-3} (.290) + .374 M_{t-4} (.289) − .026 M_{t-5} (.286) −				
		.270 M_{t-6} (.289) − .247 M_{t-7} (.293) + .087 Y_t (.166) − .041 Y_{t-1} (.167) − .035 Y_{t-2} (.165) −				
		.022 Y_{t-3} (.167) + .042 Y_{t-4} (.167) − .035 Y_{t-5} (.169) + .005 Y_{t-6} (.163) +				
		.040 Y_{t-7} (.163) + .004 $\dot{r}_t$ (.002) + .005 $\dot{r}_{t-1}$ (.002) + .006 $\dot{r}_{t-2}$ (.002) + .008 $\dot{r}_{t-3}$ (.002) +				
		.010 $\dot{r}_{t-4}$ (.002) + .010 $\dot{r}_{t-5}$ (.002) + .008 $\dot{r}_{t-6}$ (.002) + .005 $\dot{r}_{t-7}$ (.002)	.96	1.2E-4	.0001	.35

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$t{+}12\pi^e$	=	$\underset{(.414)}{1.038}P_t + \underset{(.411)}{1.319}P_{t-1} + \underset{(.412)}{1.209}P_{t-2} + \underset{(.408)}{1.144}P_{t-3} + \underset{(.408)}{.860}P_{t-4} +$				
		$\underset{(.421)}{.961}P_{t-5} + \underset{(.410)}{.957}P_{t-6} + \underset{(.413)}{.592}P_{t-7} + \underset{(.311)}{1.328}M_t + \underset{(.310)}{1.217}M_{t-1} +$				
		$\underset{(.317)}{.933}M_{t-2} + \underset{(.318)}{.399}M_{t-3} + \underset{(.318)}{.291}M_{t-4} + \underset{(.315)}{.079}M_{t-5} -$				
		$\underset{(.320)}{.408}M_{t-6} - \underset{(.322)}{.139}M_{t-7} + \underset{(.184)}{.113}Y_t + \underset{(.184)}{.045}Y_{t-1} - \underset{(.182)}{.077}Y_{t-2} +$				
		$\underset{(.183)}{.047}Y_{t-3} + \underset{(.184)}{.072}Y_{t-4} - \underset{(.186)}{.046}Y_{t-5} - \underset{(.179)}{.016}Y_{t-6} +$				
		$\underset{(.180)}{.104}Y_{t-7} + \underset{(.002)}{.004}\dot{r}_t + \underset{(.002)}{.005}\dot{r}_{t-1} + \underset{(.002)}{.007}\dot{r}_{t-2} + \underset{(.002)}{.008}\dot{r}_{t-3} +$				
		$\underset{(.002)}{.010}\dot{r}_{t-4} + \underset{(.002)}{.010}\dot{r}_{t-5} + \underset{(.002)}{.008}\dot{r}_{t-6} + \underset{(.002)}{.005}\dot{r}_{t-7}$	.95	1.4E-4	.0001	.31
$t{+}13\pi^e_t$	=	$\underset{(.1006)}{1.133}P_t + \underset{(1.001)}{1.532}P_{t-1} + \underset{(.984)}{1.429}P_{t-2} + \underset{(.970)}{1.246}P_{t-3} + \underset{(.970)}{.742}P_{t-4} +$				
		$\underset{(1.009)}{1.358}P_{t-5} + \underset{(.994)}{1.602}P_{t-6} + \underset{(1.006)}{1.160}P_{t-7} + \underset{(.732)}{2.579}M_t + \underset{(.731)}{2.085}{}_{t-1} +$				
		$\underset{(.747)}{1.877}M_{t-2} + \underset{(.755)}{.894}M_{t-3} + \underset{(.752)}{1.017}M_{t-4} + \underset{(.743)}{.671}M_{t-5} +$				
		$\underset{(.751)}{.343}M_{t-6} + \underset{(.760)}{.921}M_{t-7} + \underset{(.429)}{.522}Y_t + \underset{(.430)}{.310}Y_{t-1} + \underset{(.428)}{.310}Y_{t-2} +$				
		$\underset{(.431)}{.384}Y_{t-3} + \underset{(.431)}{.408}Y_{t-4} + \underset{(.438)}{.277}Y_{t-5} + \underset{(.419)}{.154}Y_{t-6} +$				
		$\underset{(.420)}{.239}Y_{t-7} + \underset{(.004)}{.004}\dot{r}_t + \underset{(.005)}{.007}\dot{r}_{t-1} + \underset{(.005)}{.008}\dot{r}_{t-2} + \underset{(.005)}{.008}\dot{r}_{t-3} +$				
		$\underset{(.005)}{.011}\dot{r}_{t-4} + \underset{(.005)}{.010}\dot{r}_{t-5} + \underset{(.005)}{.006}\dot{r}_{t-6} + \underset{(.004)}{.004}\dot{r}_{t-7}$	.94	7.6E-4	.0001	.15

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$t{+}36\pi^e_t$	=	$\underset{(1.448)}{.423\ P_t} + \underset{(.1429)}{1.044\ P_{t-1}} + \underset{(1.380)}{1.009\ P_{t-2}} + \underset{(1.371)}{1.082\ P_{t-3}} + \underset{(1.377)}{1.071\ P_{t-4}} +$				
		$\underset{(1.439)}{2.205\ P_{t-5}} + \underset{(1.439)}{3.086\ P_{t-6}} + \underset{(1.478)}{2.004\ P_{t-7}} + \underset{(1.044)}{3.112\ M_t} + \underset{(1.055)}{2.240\ M_{t-1}} +$				
		$\underset{(1.078)}{2.337\ M_{t-2}} + \underset{(1.095)}{1.1667\ M_{t-3}} + \underset{(1.081)}{2.160\ M_{t-4}} + \underset{(1.079)}{1.819\ M_{t-5}} +$				
		$\underset{(1.099)}{1.255\ M_{t-6}} + \underset{(1.123)}{2.203\ M_{t-7}} + \underset{(.604)}{.859\ Y_t} + \underset{(.605)}{.763\ Y_{t-1}} + \underset{(.602)}{.930\ Y_{t-2}} +$				
		$\underset{(.0608)}{.913\ Y_{t-3}} + \underset{(.607)}{.818\ Y_{t-4}} + \underset{(.625)}{.592\ Y_{t-5}} + \underset{(.594)}{.272\ Y_{t-6}} +$				
		$\underset{(.607)}{.387\ Y_{t-7}} - \underset{(.006)}{.002\ \dot{r}_t} - \underset{(.007)}{.002\ \dot{r}_{t-1}} - \underset{(.007)}{.001\ \dot{r}_{t-2}} - \underset{(.007)}{.001\ \dot{r}_{t-3}} +$				
		$\underset{(.007)}{.003\ \dot{r}_{t-4}} + \underset{(.007)}{.001\ \dot{r}_{t-5}} - \underset{(.007)}{.003\ \dot{r}_{t-6}} - \underset{(.006)}{.001\ \dot{r}_{t-7}}$	.95	1.5E-3	.0001	.13
$t{+}48\pi^e_t$	=	$\underset{(1.893)}{-.394\ P_t} + \underset{(1.890)}{.802\ P_{t-1}} + \underset{(1.809)}{1.296\ P_{t-2}} + \underset{(1.789)}{1.634\ P_{t-3}} + \underset{(1.820)}{2.310\ P_{t-4}} +$				
		$\underset{(1.858)}{3.393\ P_{t-5}} + \underset{(1.872)}{4.655\ P_{t-6}} + \underset{(1.899)}{3.609\ P_{t-7}} + \underset{(1.371)}{2.961\ M_t} +$				
		$\underset{(1.372)}{2.087\ M_{t-1}} + \underset{(1.392)}{2.498\ M_{t-2}} + \underset{(1.415)}{2.069\ M_{t-3}} + \underset{(1.393)}{2.450\ M_{t-4}} +$				
		$\underset{(1.390)}{2.056\ M_{t-5}} + \underset{(1.407)}{1.685\ M_{t-6}} + \underset{(1.402)}{2.851\ M_{t-7}} + \underset{(.774)}{1.287\ Y_t} + \underset{(.775)}{1.359\ Y_{t-1}} +$				
		$\underset{(.777)}{1.540\ Y_{t-2}} + \underset{(.776)}{1.636\ Y_{t-3}} + \underset{(.775)}{1.354\ Y_{t-4}} + \underset{(.789)}{1.375\ Y_{t-5}} +$				
		$\underset{(.755)}{.844\ Y_{t-6}} + \underset{(.759)}{.851\ Y_{t-7}} - \underset{(.008)}{.011\ \dot{r}_t} - \underset{(.009)}{.012\ \dot{r}_{t-1}} - \underset{(.009)}{.011\ \dot{r}_{t-2}} -$				
		$\underset{(.009)}{.013\ \dot{r}_{t-3}} - \underset{(.009)}{.006\ \dot{r}_{t-4}} - \underset{(.010)}{.009\ \dot{r}_{t-5}} - \underset{(.010)}{.010\ \dot{r}_{t-6}} - \underset{(.009)}{.007\ \dot{r}_{t-7}}$	.95	2.2E-3	.0001	.17

Table 1 Cont'd

Monthly Horizon t+n		Equation	$\bar{R}^2$	MSE	Prob > F	DW
$_{t+60}\pi^e_t$	=	$\underset{(1.976)}{1.028} P_t + \underset{(1.986)}{2.388} P_{t-1} + \underset{(1.980)}{3.086} P_{t-2} + \underset{(1.914)}{4.202} P_{t-3} + \underset{(2.005)}{4.681} P_{t-4} +$				
		$\underset{(2.250)}{5.947} P_{t-5} + \underset{(2.286)}{6.502} P_{t-6} + \underset{(2.255)}{6.946} P_{t-7} + \underset{(1.382)}{2.578} M_t + \underset{(1.378)}{2.321} M_{t-1} +$				
		$\underset{(1.403)}{2.703} M_{t-2} + \underset{(1.421)}{1.963} M_{t-3} + \underset{(1.399)}{2.406} M_{t-4} + \underset{(1.388)}{1.810} M_{t-5} +$				
		$\underset{(1.403)}{1.630} M_{t-6} + \underset{(1.412)}{2.665} M_{t-7} + \underset{(.797)}{1.303} Y_t + \underset{(.792)}{1.596} Y_{t-1} + \underset{(.793)}{1.995} Y_{t-2} +$				
		$\underset{(.784)}{2.197} Y_{t-3} + \underset{(.786)}{1.943} Y_{t-4} + \underset{(.807)}{1.821} Y_{t-5} + \underset{(.782)}{1.036} Y_{t-6} +$				
		$\underset{(.775)}{.497} Y_{t-7} - \underset{(.009)}{.018} \dot{r}_t - \underset{(.010)}{.018} \dot{r}_{t-1} - \underset{(.010)}{.018} \dot{r}_{t-2} - \underset{(.010)}{.019} \dot{r}_{t-3} -$				
		$\underset{(.010)}{.014} \dot{r}_{t-4} - \underset{(.010)}{.013} \dot{r}_{t-5} - \underset{(.011)}{.008} \dot{r}_{t-6} - \underset{(.010)}{.006} \dot{r}_{t-7}$	.97	2.1E-3	.0001	.27

The Main Test

The best scheme for each horizon of expected inflation having been formulated in the preliminary step, interest rates of different maturities were then regressed on the various annualized horizons of Table 1 using a first order autoregressive scheme

$$r_t = \alpha_o + \beta_1 \; {}_{t+n}\pi_t^e + \varepsilon_t \tag{39}$$

where

$$\varepsilon_t = \mu_t - \alpha_1 \; \varepsilon_{t-1} \tag{40}$$

and μ_t is normally and independently distributed with mean 0 and variance σ^2.[2] We should expect the better fit for inflation expectations to cause the estimated value beta, β_1, to approach in value the true value of beta. If we expect the true value of beta to be one, then we should observe the values of beta increasing towards the value one as we substitute the better expectation fits into the equation. The true value of beta is unknown. It should be noted that it is possible for movement to be away from the true value. Table 2 is the first order autoregression of interest rates on the annualized inflation schemes from Table 1.

Variables which were placed in the expectations formulation are then placed into the interest rate equation as additional variables, z_t, control variables. Thus

following the same procedure as described above,

$$r_t = \alpha_o + \beta_1 \; {}_{t+1}\pi_t^e + \varepsilon_t \tag{41}$$

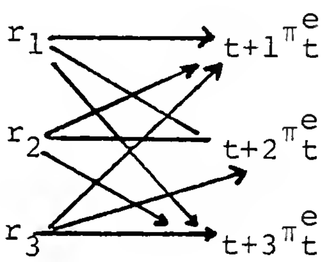
is compared with

$$r_t = \alpha_o + \beta_1 \; {}_{t+1}\pi_t^e + \beta_2 \; M_t + \beta_3 \; Y_t + \beta_4 \; \dot{r}_{t-1} + \varepsilon_t \tag{42}$$

to conclude if any additional explanatory power can be gained with the added determinates. The test is then repeated for each interest rate. Table 3 gives the results of equation (42) for a first order autoregression.

Thus the horizons are mixed according to the following pattern.

Pattern A



etc.

If the regressions show that the expected inflation formulated for a given maturity best explains the interest rate of that maturity, then the pattern for the highest $\bar{R}^2$ observed would be that of the following,

Table 2
The Fisher Equation, 1/59 to 12/78
Beta/R Square Values

INTEREST RATES, r_t \ Inflationary Expectations $_{t+n}\pi^e_t$	1 mo.	2 mo.	3 mo.	4 mo.	5 mo.	6 mo.	7 mo.	8 mo.	9 mo.
1 mo.	.35/.37	.34/.31	.35/.34	.37/.35	.40/.41	.42/.44	.43/.45	.44/.41	.45/.41
2 mo.	.32/.35	.31/.29	.32/.32	.33/.33	.36/.37	.38/.40	.38/.40	.40/.35	.40/.35
3 mo.	.31/.34	.30/.28	.31/.31	.33/.32	.35/.36	.37/.38	.37/.38	.38/.32	.38/.32
4 mo.	.32/.35	.31/.30	.32/.32	.34/.34	.37/.38	.38/.41	.38/.40	.39/.34	.39/.34
5 mo.	.33/.36	.32/.30	.33/.33	.34/.35	.37/.39	.39/.41	.38/.40	.39/.33	.39/.33
6 mo.	.33/.36	.31/.30	.33/.33	.34/.34	.37/.39	.39/.41	.38/.40	.39/.33	.39/.33
12 mo.	.41/.37	.43/.36	.43/.37	.45/.39	.46/.42	.47/.43	.46/.44	.46/.42	.47/.42
24 mo.	.30/.36	.29/.31	.31/.36	.31/.36	.32/.37	.32/.37	.29/.31	.27/.20	.27/.19
36 mo.	.29/.35	.28/.30	.30/.35	.30/.35	.30/.35	.29/.35	.26/.27	.24/.16	.23/.15
48 mo.	.27/.33	.27/.29	.29/.34	.29/.34	.28/.33	.28/.33	.24/.24	.22/.14	.21/.13
60 mo.	.26/.32	.25/.28	.28/.34	.28/.33	.27/.32	.26/.31	.22/.22	.19/.12	.18/.11

Table 2 Cont'd

INTEREST RATES, r_t \ Inflationary Expectations, $_{t+n}\pi^e_t$	10 mo.	11 mo.	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
1 mo.	.45/.41	.46/.41	.46/.39	.41/.19	.38/.13	.40/.14	.44/.20
2 mo.	.41/.35	.41/.34	.41/.33	.36/.16	.34/.12	.34/.12	.38/.18
3 mo.	.39/.32	.39/.31	.39/.30	.33/.14	.30/.09	.31/.10	.36/.16
4 mo.	.40/.33	.40/.32	.40/.31	.34/.15	.32/.10	.33/.11	.38/.18
5 mo.	.39/.33	.39/.32	.40/.31	.34/.15	.31/.10	.32/.11	.36/.17
6 mo.	.40/.33	.40/.32	.40/.30	.35/.16	.32/.11	.32/.11	.37/.17
12 mo.	.48/.42	.48/.42	.49/.42	.53/.33	.51/.23	.50/.23	.46/.30
24 mo.	.28/.20	.28/.19	.29/.20	.29/.14	.28/.11	.30/.13	.34/.20
36 mo.	.24/.16	.24/.15	.25/.16	.27/.13	.28/.11	.31/.14	.34/.23
48 mo.	.22/.13	.22/.13	.23/.13	.25/.12	.27/.11	.30/.15	.34/.23
60 mo.	.19/.11	.19/.11	.20/.11	.23/.11	.26/.11	.29/.15	.33/.24

Table 3
Multiple Regression, 1/59 to 12/78
Beta_1/R Square Values

Inflationary Expectations $_{t+n}\pi_t^e$

Interest Rates, r_t	1 mo.	2 mo.	3 mo.	4 mo.	5 mo.	6 mo.	7 mo.	8 mo.	9 mo.
1 mo.	.39/.43	.40/.41	.42/.45	.44/.48	.46/.52	.47/.54	.47/.54	.49/.53	.49/.53
2 mo.	.36/.41	.38/.40	.39/.44	.41/.46	.43/.50	.44/.52	.44/.51	.46/.49	.46/.48
3 mo.	.36/.41	.37/.40	.39/.44	.41/.46	.43/.49	.44/.51	.43/.50	.45/.47	.45/.46
4 mo.	.37/.42	.38/.41	.40/.45	.42/.48	.44/.52	.45/.53	.44/.52	.46/.48	.46/.47
5 mo.	.38/.43	.39/.42	.41/.46	.43/.49	.45/.53	.45/.54	.44/.52	.45/.47	.45/.47
6 mo.	.38/.43	.39/.42	.41/.46	.43/.49	.45/.53	.46/.54	.45/.52	.45/.47	.45/.46
12 mo.	.41/.38	.43/.37	.43/.39	.45/.41	.46/.43	.47/.44	.46/.44	.46/.42	.47/.42
24 mo.	.32/.39	.33/.37	.35/.42	.35/.42	.35/.43	.35/.42	.32/.34	.30/.24	.30/.23
36 mo.	.29/.36	.30/.34	.32/.39	.32/.38	.31/.38	.31/.37	.27/.28	.26/.18	.25/.17
48 mo.	.28/.34	.28/.32	.30/.36	.30/.36	.29/.34	.28/.33	.24/.24	.23/.15	.22/.14
60 mo.	.26/.33	.27/.30	.28/.35	.28/.34	.27/.32	.26/.31	.22/.21	.20/.12	.19/.11

Table 3 Cont'd

INTEREST RATES, r_t	Inflationary Expectations, ${}_{t+n}\pi^e_t$						
	10 mo.	11 mo.	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
1 mo.	.50/.52	.51/.52	.51/.51	.52/.36	.49/.26	.50/.26	.51/.36
2 mo.	.47/.48	.47/.48	.48/.46	.49/.34	.46/.24	.45/.25	.47/.33
3 mo.	.46/.46	.46/.46	.47/.45	.47/.32	.44/.23	.44/.23	.46/.32
4 mo.	.47/.47	.47/.47	.48/.46	.49/.34	.45/.24	.45/.25	.47/.34
5 mo.	.46/.47	.47/.46	.47/.45	.48/.33	.45/.24	.45/.25	.47/.34
6 mo.	.46/.46	.47/.46	.47/.45	.49/.34	.46/.25	.46/.25	.48/.35
12 mo.	.48/.42	.48/.42	.49/.43	.53/.35	.53/.26	.52/.26	.48/.34
24 mo.	.31/.24	.32/.24	.33/.24	.38/.24	.39/.22	.42/.25	.43/.35
36 mo.	.26/.17	.27/.17	.28/.18	.34/.19	.36/.20	.40/.24	.41/.33
48 mo.	.23/.14	.23/.14	.24/.14	.31/.17	.34/.18	.38/.23	.39/.31
60 mo.	.20/.12	.20/.11	.22/.12	.28/.15	.33/.18	.37/.23	.38/.30

Pattern B

$$r_1 \longrightarrow {}_{t+1}\pi^e_t$$

$$r_2 \longrightarrow {}_{t+2}\pi^e_t$$

$$r_3 \longrightarrow {}_{t+3}\pi^e_t$$

etc.

rather than Pattern A, or some scheme similar to that of Pattern A. Thus if Pattern B is observed we can conclude that the market rate of interest on debt of different durations could best be explained by inflationary expectations over different horizons that correspond to those debt horizons.

The Subperiod Test

A subsequent test was performed for the time period January 1965 to December 1978 using the autoregressive schemes developed from the main period, January 1959 to December 1978. The subperiod testing allowed the inclusion of seven to eleven month T-bills in the output results in addition to the purpose of studying the results for consistency during a period of rapid inflation. The yearly mean inflation rate was larger in the subsequent period, changing from an arithmetic mean of 4.3 in the whole period to an arithmetic mean of 5.7 in the subperiod.

Tables 4 and 5 contain the results for the subperiod January 1965 to December 1978 using the inflationary expectations formulation indicated on Table 1 for Tables 2 and 3.

From initial data screens on the subperiod it was obvious that this was a period where formulation of inflationary expectations had changed. Thus some similarity of results in a period with different autoregressive orders and rapid inflation would be taken as additional confidence in the original results.

Notes

1. Data for Gross Personal Income and Money Supply, M1, are seasonally adjusted monthly observations from the November 1979 CITIBASE tape compiled and updated by Citicorp National Bank, New York, and supplied courtesy of the Bureau of Business and Economic Research, University of Florida, Gainesville. Data for Consumer Price Index was supplied courtesy of the Bureau of Labor Statistics, Department of Commerce, Washington, D.C. Data for T-bills are discount yields (bid yields) as of the end of the preceding month from Salomon Brothers quote sheets. The bid yields are converted to bond equivalents by the following calculation

$$r(\text{semiannual}) = 2\left[\left(\frac{1}{1-\frac{dxt}{360}}\right)^{365/2t} - 1\right]$$

where the discount rate on a 360-day T-bill is converted to a semiannual compounded rate based on a 365-day year, r represents the semiannual interest rate, d represents the annual discount rate, and t represents the time in days til maturity. Bond data are beginning of the month series of yields of United

Table 4
The Fisher Equation, 1/65 to 12/78
Beta/R Square Values

INTEREST RATES, r_t \ Inflationary Expectations, $_{t+n}\pi^e_t$	1 mo.	2 mo.	3 mo.	4 mo.	5 mo.	6 mo.	7 mo.	8 mo.	9 mo.
1 mo.	.22/.18	.23/.15	.22/.14	.23/.14	.26/.17	.27/.19	.32/.24	.34/.23	.35/.23
2 mo.	.21/.17	.22/.16	.21/.15	.21/.14	.23/.16	.24/.17	.29/.23	.30/.19	.30/.19
3 mo.	.20/.16	.22/.16	.20/.14	.21/.13	.23/.15	.23/.16	.28/.20	.29/.17	.29/.17
4 mo.	.21/.18	.22/.17	.21/.15	.23/.16	.25/.18	.26/.19	.30/.24	.31/.19	.31/.19
5 mo.	.22/.19	.23/.17	.22/.16	.23/.16	.25/.19	.26/.20	.30/.24	.31/.20	.31/.20
6 mo.	.22/.20	.24/.18	.22/.16	.23/.16	.26/.19	.26/.20	.31/.25	.31/.20	.31/.19
7 mo.	.23/.21	.24/.19	.22/.16	.23/.16	.26/.19	.26/.20	.31/.25	.30/.19	.30/.19
8 mo.	.23/.21	.24/.18	.23/.16	.24/.16	.26/.20	.27/.20	.30/.24	.30/.10	.30/.18
9 mo.	.24/.23	.24/.20	.22/.18	.23/.17	.25/.20	.26/.21	.30/.26	.29/.18	.29/.17
10 mo.	.25/.24	.25/.21	.22/.18	.23/.18	.25/.21	.26/.22	.30/.27	.29/.17	.28/.16
11 mo.	.25/.24	.24/.21	.22/.19	.23/.18	.25/.21	.26/.22	.30/.27	.28/.17	.27/.16
12 mo.	.31/.19	.32/.17	.30/.15	.32/.17	.35/.20	.36/.21	.35/.23	.36/.23	.36/.23
24 mo.	.17/.16	.18/.14	.18/.15	.19/.14	.20/.17	.20/.17	.21/.16	.20/.11	.19/.10
36 mo.	.15/.13	.16/.13	.17/.14	.17/.13	.18/.15	.10/.15	.18/.13	.18/.09	.16/.07
48 mo.	.14/.11	.15/.11	.15/.12	.16/.12	.16/.13	.16/.13	.16/.10	.16/.07	.14/.06
60 mo.	.12/.10	.13/.10	.14/.11	.14/.11	.14/.11	.14/.11	.13/.08	.14/.06	.12/.04

Table 4 Cont'd

INTEREST RATES, r_t	Inflationary Expectations $_{t+n}\pi^e_t$: 10 mo.	11 mo.	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
1 mo.	.35/.23	.35/.22	.36/.21	.28/.07	.28/.06	.31/.07	.37/.12
2 mo.	.32/.20	.31/.18	.32/.18	.26/.07	.26/.06	.27/.06	.32/.09
3 mo.	.30/.17	.29/.16	.30/.16	.22/.05	.20/.03	.23/.04	.30/.08
4 mo.	.32/.19	.31/.18	.32/.18	.25/.06	.24/.05	.26/.06	.34/.10
5 mo.	.32/.20	.31/.19	.32/.18	.24/.06	.22/.04	.23/.05	.30/.08
6 mo.	.32/.20	.32/.19	.33/.19	.26/.07	.24/.05	.24/.05	.31/.09
7 mo.	.32/.19	.31/.18	.33/.18	.26/.07	.24/.05	.24/.05	.29/.08
8 mo.	.31/.19	.31/.18	.32/.17	.26/.07	.24/.05	.24/.05	.29/.08
9 mo.	.32/.19	.31/.17	.33/.18	.26/.07	.23/.05	.21/.04	.25/.07
10 mo.	.31/.18	.30/.17	.32/.17	.25/.07	.22/.04	.21/.04	.26/.07
11 mo.	.31/.18	.29/.16	.32/.17	.26/.07	.22/.04	.20/.04	.25/.07
12 mo.	.37/.23	.37/.23	.38/.23	.39/.12	.30/.04	.25/.03	.29/.07
24 mo.	.21/.11	.21/.10	.23/.11	.22/.07	.19/.04	.20/.04	.24/.08
36 mo.	.18/.08	.17/.08	.20/.09	.20/.06	.19/.04	.21/.05	.25/.09
48 mo.	.16/.07	.15/.06	.18/.07	.19/.06	.19/.05	.21/.06	.24/.09
60 mo.	.14/.06	.13/.05	.15/.06	.17/.05	.18/.05	.20/.06	.23/.09

Table 5
Multiple Regression, 1/65 to 12/78
$Beta_1$/R Square Values

Interest Rates, r_t \ Inflationary Expectations, $_{t+n}\pi^e_t$	1 mo.	2 mo.	3 mo.	4 mo.	5 mo.	6 mo.	7 mo.	8 mo.	9 mo.
1 mo.	.29/.29	.31/.29	.31/.29	.34/.31	.36/.36	.38/.37	.40/.41	.43/.43	.43/.43
2 mo.	.27/.28	.30/.28	.30/.29	.32/.30	.34/.34	.35/.35	.38/.39	.40/.39	.40/.38
3 mo.	.27/.27	.30/.29	.30/.28	.32/.29	.34/.33	.35/.34	.37/.37	.39/.38	.39/.37
4 mo.	.28/.30	.31/.30	.31/.30	.33/.32	.36/.36	.36/.37	.39/.40	.40/.40	.40/.39
5 mo.	.29/.31	.31/.30	.31/.31	.33/.32	.36/.37	.37/.38	.39/.41	.40/.39	.40/.38
6 mo.	.29/.31	.32/.31	.32/.31	.34/.33	.36/.37	.37/.38	.39/.41	.40/.39	.40/.38
7 mo.	.29/.32	.32/.31	.32/.31	.33/.32	.36/.37	.36/.37	.38/.40	.39/.37	.39/.37
8 mo.	.30/.32	.32/.31	.32/.31	.34/.33	.36/.37	.36/.38	.38/.40	.38/.36	.38/.35
9 mo.	.30/.34	.32/.33	.30/.31	.32/.33	.35/.37	.35/.37	.37/.40	.37/.34	.37/.33
10 mo.	.30/.34	.32/.33	.30/.32	.32/.33	.35/.37	.35/.37	.37/.39	.37/.33	.36/.31
11 mo.	.31/.36	.32/.35	.30/.33	.32/.34	.34/.38	.35/.38	.36/.40	.36/.33	.35/.31
12 mo.	.31/.21	.31/.20	.30/.18	.32/.20	.35/.23	.35/.24	.35/.25	.35/.25	.35/.25
24 mo.	.21/.22	.23/.22	.23/.23	.24/.23	.25/.25	.24/.25	.24/.23	.25/.18	.23/.16
36 mo.	.17/.17	.19/.17	.20/.19	.20/.19	.21/.19	.20/.19	.20/.16	.20/.12	.18/.11
48 mo.	.15/.13	.17/.14	.17/.16	.17/.15	.18/.16	.17/.15	.16/.12	.17/.09	.15/.08
60 mo.	.13/.11	.15/.12	.15/.13	.16/.13	.15/.13	.15/.13	.14/.09	.15/.07	.13/.06

Table 5 Cont'd

Interest Rates, r_t	Inflationary Expectations, $_{t+n}\pi_t^e$: 10 mo.	11 mo.	12 mo.	24 mo.	36 mo.	48 mo.	60 mo.
1 mo.	.44/.42	.44/.41	.45/.40	.41/.20	.34/.13	.36/.14	.43/.21
2 mo.	.41/.38	.41/.37	.42/.36	.39/.19	.32/.12	.32/.12	.39/.19
3 mo.	.40/.37	.41/.36	.41/.35	.37/.17	.28/.10	.29/.10	.38/.18
4 mo.	.41/.39	.42/.38	.42/.37	.38/.19	.30/.12	.31/.12	.40/.20
5 mo.	.41/.38	.41/.37	.42/.36	.38/.18	.30/.11	.31/.11	.39/.19
6 mo.	.41/.38	.41/.37	.42/.36	.39/.19	.31/.12	.32/.12	.40/.20
7 mo.	.40/.37	.40/.36	.41/.35	.38/.18	.32/.11	.32/.12	.40/.20
8 mo.	.40/.35	.40/.34	.41/.34	.38/.18	.32/.11	.33/.12	.40/.20
9 mo.	.39/.34	.39/.32	.40/.32	.34/.15	.25/.09	.25/.09	.36/.17
10 mo.	.39/.32	.39/.31	.40/.31	.34/.15	.26/.09	.26/.09	.37/.17
11 mo.	.38/.32	.38/.31	.39/.31	.34/.16	.25/.10	.24/.10	.35/.18
12 mo.	.36/.25	.37/.25	.37/.25	.38/.14	.29/.07	.27/.06	.31/.11
24 mo.	.26/.18	.26/.17	.27/.18	.28/.13	.25/.09	.27/.10	.33/.17
36 mo.	.21/.12	.21/.11	.22/.12	.24/.09	.23/.08	.26/.10	.31/.16
48 mo.	.18/.09	.17/.08	.19/.09	.22/.08	.22/.07	.26/.09	.30/.15
60 mo.	.15/.07	.15/.06	.17/.07	.19/.07	.21/.07	.24/.09	.28/.13

States securities read from monthly yield curves prepared by Saloman Brothers. Whenever there is a choice of coupons, the curves follow the yields of higher coupon issues in the longer maturities.

2. The annualized interest rates were given, and the inflationary patterns of Table 1 were annualized by compounding each to one year as in the following example of the monthly expectation

$$ {}_{t+1}\pi_t^e(A) = \prod_{k=1}^{12} (1 + {}_{t+1}\pi_t^e) - 1 $$

likewise to annualize ${}_{t+2}\pi_t^e$

$$ {}_{t+2}\pi_t^e(A) = \prod_{k=1}^{6} (1 + {}_{t+2}\pi_t^e) - 1 \quad . $$

Thus Table 1 was annualized and regressed against annualized interest rates. Missing observations, due to differencing the data for equation fit in Table 1, were generated by the expectation equation for the respective horizon. Thus expectations values for longer horizons were "filled" in for the later years of the data set under the assumption that expectation were formulated in the same manner. Thus sample sizes are equal.

CHAPTER IV
ANALYSIS OF RESULTS

The Term Structure of Inflationary Expectations

Table 1 indicates a limited number of different inflationary patterns. The actual number of different lag patterns to be matched is six. The spread between the patterns and the change in loading patterns in Table 1 implies that individuals formulate, at a minimum, two distinct sets of inflationary expectations.

Two sets are isolated primarily by the statistical significance of the lagged inflation variables at a .10 level. In the first set, lagged values of price changes (inflation) plus various combinations of additional variables generate expectations. The second set of expectations, horizons twenty-four to fifty months, shall be called long-term expectations. In the second set, lagged values of money supply growth plus various combinations of additional variables generate expectations.

The term structure of inflationary expectations has an upward drift and indicates a four year cycle. Thus the lag pattern is unique in most of the first eight periods and tries to correct itself with additional

information from past inflation. It adjusts itself for the remainder of the term structure to the eight month lag pattern.

The relatively short pattern of lagged values is taken to indicate that individuals anticipate the longer-run impacts of changing economic conditions much quicker than in previous periods of stable monetary growth. The use of additional information from the other time series and the more active inflation rates after 1965 are offered as explanations for the change from past formations of long lag patterns.

Table 1 indicates the question, "Do changes in the inflation rate cause changes in interest rates?" is justified only as a reflection of the primary determinate of expected inflation. In the study of short-term interest rates there exists more than the one determinate, past and current price changes, for the formation of expected inflation. The study of long-term interest rates, however, is the study of the movement of both interest rates and inflation responding primarily to changes in the growth rate of the money supply.

This observation from Table 1 of the different loading patterns of short-term and long-term interest rates described above is weakened when the business cycle repeats itself. The various alternating upswings and downswings of varied length and intensity occur at slightly less than

forty-eight months. The observation also is weakened at turning points, directional changes, within the business cycle. This marked difference in loading on interest rates at turning points suggests a new way to forecast phases of the cycle.

For example, the twenty-four month inflationary expectations are influenced not only by the growth rate of the money supply, but are influenced also by the past change in one month interest rates. Expectations for the fourth year repeat the business cycle and are influenced by income changes, as well as, money supply growth. Determinates for the five year forecast are a similar combination of the four year determinates and the one year determinates. This is taken to indicate that the individual is forecasting that five years from the present time of forecast the business cycle will be one year into repeating another second cycle.

Table 1 shows the assumption made in Chapter 2 concerning the convergence of information to a single series, the price series, is unjustified.

The following conclusions concerning expectations formulation were reached. There exist two distinctly different sets of inflationary expectations. Each set of expectations is generated by a different stochastic process. Expectations of inflation are influenced by the use of information other than the price series. The market place

takes into consideration not only the current and past values of inflation to predict the future course of inflation, but also uses information concerning the rate of growth of personal income, the growth rate of the money supply, and past changes in the one-month interest rate.

Interest Rates and Inflationary Expectations

To answer the question, "How much of the structure of interest rates is caused by the structure of expected inflation?" two types of evidence are presented. First, an examination of summary statistics and graphs is presented to determine whether there is a structure to expected inflation which is in some way comparable to the structure of interest rates. Second, the major results of matching interest rate maturities and expectations horizons are presented for the different time periods and different interest rate equations to determine the matching with the highest explanatory power.

Summary Statistics and Graphs

The information presented in Table 6 and Figure 1 is designed to answer the following two questions: "Do the π^e's vary as much across the horizons as the r's vary across maturities?" and "Does the expectation "yield curve" bend up and down when the interest rate curve does?"

Going vertically down Table 6 the mean r's rise and the variances of the r's fall. This is as expected. The

same is true of the π^e's at least for the variances; the means behave slightly different by having only a general upward trend. However, the behavior of the mean π^e's does not damage the theory. Only if the variances indicated a different trend pattern would the theory be damaged.

Table 6
Summary Statistics for
Interest Rates and Inflationary Expectations
1/59 to 12/78

Horizon in months	r mean μ_{r_t}	Standard Deviation $\sqrt{\sigma^2_{r_t}}$	π^e_t mean $\mu_{\pi^e_t}$	Standard Deviation $\sqrt{\sigma^2_{\pi^e_t}}$
1	4.61	1.78	4.50	2.73
2	4.70	1.68	4.54	2.66
3	4.90	1.67	4.56	2.61
4	4.90	1.67	4.56	2.61
5	4.98	1.67	4.56	2.64
6	5.04	1.66	4.57	2.60
12	5.21	1.75	4.53	2.56
24	5.44	1.63	4.56	2.13
36	5.56	1.58	4.56	2.01
48	5.64	1.55	4.58	1.97
60	5.69	1.55	4.59	1.95

Figure 1 helps to highlight the results of Table 6. It adds evidence that the two structures are comparable. The π^e's do vary as much across horizons as the r's vary across maturities. As the expectations "yield curve" bends up the interest rate curve bends up, and as the expectations "yield curve" bends down the interest rate curve does also. Interesting is the observation of increased frequency after 1968 of the times the portion

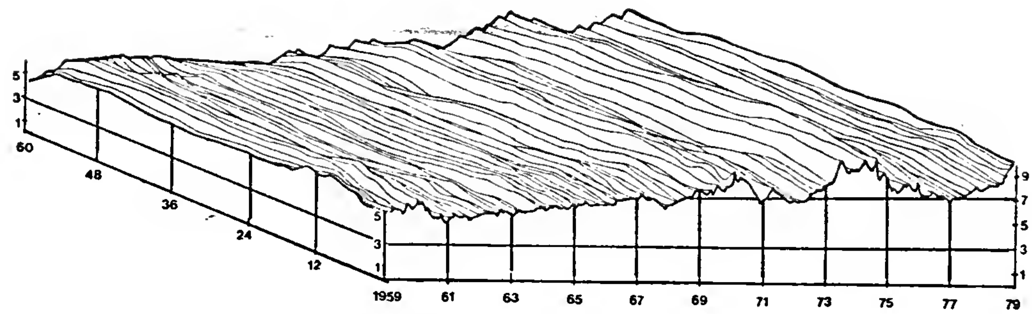


Figure 1 - A. The Term Structure of Interest Rates

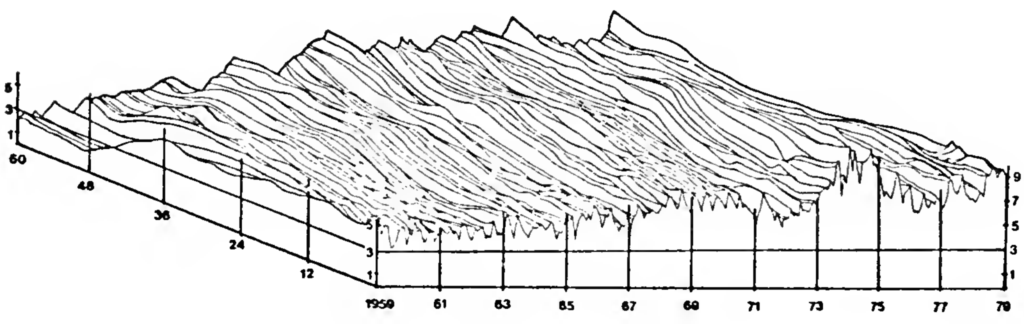


Figure 1 - B. The Term Structure of Inflationary Expectations

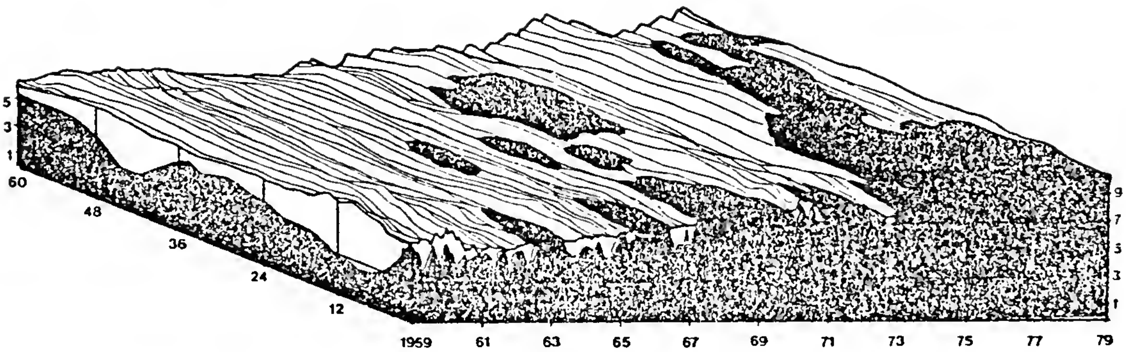


Figure 1 - C. Inflationary Expectation and the Term Structure of Interest Rates

of short-term expectations curve lies above the interest rate curve. This is interpreted as the result of the increased inflation activity and more rapid information processing by the market place.

The above results from Table 6 and Figure 1 shows the structure of expected inflation is comparable to the structure of interest rates.

<u>Maturity Matching</u>

Tables 2 and 4 represent the various interest rates regressed on the different inflation horizons with an adjustment for first order serial correlation for the Fisher equation. Tables 3 and 5 show the same results for the multivariate regression model. Therefore the result of the existence of at least two distinct horizon was not model sensitive. However, the results of the loading patterns were sensitive to the rate of inflation, i.e., the main period or the subperiod, and the specification of a single variable or multiple variable interest rate equation. The results below show the highest loadings during the main period. Thus if horizons do matter, the pattern of loadings exhibited in the main period imply that the horizons of the interest rates does not match that of the expected inflation rate as a unique set.

Interest Rates (monthly maturities)	Inflationary Expectations Table 2 (annualized horizon)	Inflationary Expectations Table 3 (annualized horizon)
1	7	6,7
2	6,7	6
3	6,7	6
4	6	6
5	6	6
6	6	6
12	7	6
24	6,6	5
36	1,3,4,5,6	3
48	3,4	3,4
60	3	3

Likewise the loading patterns for the subperiod of more rapid inflation indicate

Interest Rates (monthly maturities)	Inflationary Expectations Table 2 (annualized horizons)	Inflationary Expectations Table 3 (annualized horizons)
1	7	8,9
2	7	7,8
3	7	8
4	7	7,8
5	7	7
6	7	7
7	7	7
8	7	7
9	7	7
10	7	7
11	7	7
12	7,8,9,10,11,12	7,8,9,10,11,12
24	5,6	5,6
36	5,6	3,4,5,6
48	5,6	3,5
60	3,4,5,6	3,4,5,6

Again if horizons do matter, i.e., the small changes in the R^2 values are thought to imply some significance, then the pattern of loadings does not support the Fisher hypothesis.

Information processing by the market place concerning inflationary expectations shows little if any real difference between matches in short-term rates or the long-term rates. The major difference in loading patterns appears to be between the two groups short and long rates.

It appears that individuals forecast the 6 month and 7 month inflation rate very well, and that they use these two durations as the basis for most of the interest rate equations.

At no time with the use of any of the different models did the nominal interest rate fully adjust to expected inflation. The sign of the price expectations influence was always positive and slightly sensitive to the use of a single series or a multiple series expectation formulation. This is not taken as a rejection of the Fisher hypothesis. It is taken as an indication that the Fisher equation will hold only during periods that occasion conditions of long-run classical equilibrium. Therefore, if the data are during periods of transition, the Fisher hypothesis is not expected to empirically hold for the duration of debt instruments used. The more rapid the rate of price increase, the weaker will be the Fisher relationship between expected inflation and nominal interest rates. This is easily seen in the comparison of Table 2 of the main period, 1/59 to 12/78, to Table 4 of the subperiod, 1/65 to 12/78.

Likewise, the comparison of Table 3 to Table 5, the more rapid the rate of price increase the more valuable the change in the money supply became in explaining the nominal interest rate. The negative sign and strength of money indicates the strong influence the "liquidity effect" (Friedman 1968) had on nominal interest rates during the period of rapid inflation for interest rates with a 11 month or less horizon.

The statistical loss of significance for the "real output" proxy, personal income, from the intermediate and long-term interest rates, from Table 3 to Table 5 is interpreted as a result of the disturbance from monetary sources, M_t.

The increase in parameter values of ${}_{t+n}\pi_t^e$ from Tables 2 and 4 to Tables 3 and 5, respectively, and the increase in $\bar{R}^2$ value is taken to imply the correct specification of interest rate determinates in Tables 2 and 4 had been relegated to the disturbance term. That is, a multivariate model is more desirable in explaining nominal interest rates than a single variable model. This specification was sensitive to the duration of the rates. Thus both the determinates of the interest rate and the results of the Fisher equation "test" are sensitive to sample period choice and duration of rates chosen.

It is interesting that the general matching pattern increases in maturity as the subperiod loadings are

compared to the main period. Thus individuals would need to extend their expectations horizon during periods of rapid inflation in order to better explain interest rate movements. This maturity extension, due to the increased uncertainty of the holding (opportunity) cost of money, implies expected inflation will underestimate actual inflation. The six month T-bill should have reflected within its "price" the seven month expected inflation instead of the six month expectations. Interest rate forecast with matched horizons will also tend to be nonoptimal.

The final point to be noted is the reduction in MSE of the interest rate with the use of the multivariable model. If the MSE is considered as a proxy for the variance, and the variance is the appropriate measure of risk for the purchase of an investment (or a portfolio of investments), then the reduction in risk, and the measure of performance by a risk manager, can be improved with the additional variables in the interest rate equation, especially during periods of rapid inflation.

Problems and Extensions

Any empirical study within the field of economics can be said to have problems, either with theory or measurement. This study is certainly no exception.

The two major problems concern the development of Table 1. First, serial correlation in the forward rates

makes use of summary statistics typically used for OLS comparison very suspect, i.e., the t statistic, the F statistic, and the MSE. Likewise, the lagged values of the L.H.S. variable on the R.H.S. invalidates the D.W. statistic except as a measure of direction for the increase or decrease of correlation. Second, the decrease in the sample size in order to fit the various expectations horizons with the actual differenced price series makes the stability of the parameters and comparison across sample sizes suspect. Generating values for missing data was justified with the assumption that for the observations lost at the end of the dataset the formation of the expectations did not change.

Timing of the data is suspect when using the C.P.I. and end-of-month interest rates to generate Fisher equations. The regressions were adjusted forward and lagged back by one month to test sensitivity. The results showed that timing did not statistically matter.

The concern with developing and analyzing an integrated macromodel, rather than with any detailed discussion of particular expenditure functions, caused the omission of devotion to detailed consumption, investment, government, or money demand functions. Thus the theoretical model is simple and permits inflationary expectations to enter into the system only through the investment function and the price adjustment process and overlooks the

elasticity with respect to the demand for money (Friedman 1956) and the proportion of government expenditure financed by increasing the monetary base (Christ 1968) and taxes.

It is suggested that if the variable on the R.H.S. meant to capture future government anticipations in the credit market were altered to using a listing of future debt sales of the government that the model could be improved.

A very interesting observation is the movement of the real rate α_0. The real rates for Tables 2, 3, 4 and 5, with the exception of the twelve month rates, increased slightly in value as the maturity of the debt increased. Because the equation was not controlled for the various effects on the real rate (Mundell 1963 and Phillips 1958 and Darby 1975 and Friedman 1977 and Feldstein and Summers 1978), the various movements in the real rate from Table 2 to Table 3, and along the horizon on the same Table 2 or 3, and the comparison with the subperiod Tables 4 and 5, must have offered as an explanation of occurrence being due to statistical modeling. However, several interesting questions arise. Does the real rate remain constant, or increase with the horizon? Does price uncertainty, measured by the variance of the forecast, cause the real rate to rise during periods of rapid inflation? Does the real rate approach a constant for the longer maturity horizons, e.g., 5 year bonds? What are the determinates

for the real rate of interest, i.e., should it be considered as a variable in the Fisher hypothesis when testing during transition periods,

$$r_t = \beta_o \rho_t + \beta_1 \pi_t^e + \varepsilon_t \quad . \tag{43}$$

Did the increase in the savings to disposable income ratio during the subperiod, from 6.2% to 6.5% shift the net savings curve to the right by a smaller dynamic growth than the shift to the right in the government deficit training, thus causing the real rate to rise? There are many possible economic and statistical answers that could be offered for the movement of the real rate. It is hoped that additional study will answer these questions.

CHAPTER V
CONCLUSIONS

The results of this study show that for the main period, 1/59 to 12/78, that the use of a multivariate model to forecast inflation both reduces the MSE of the forecast and gives a better $\bar{R}^2$ (explanatory fit) for the interest rate. Moreover, the different stochastic processes underliying the term structure of inflationary expectations, as indicated by both the different lag patterns and the different loading patterns, suggest segmentation of the expectations curve.

The analysis of the structure of inflationary expectations and the structure of interest rates indicates the two structures are comparable. A slight difference was observed between the short-term and long-term relationships. This difference in relationships was much more pronounced during periods of rapid inflation.

It was shown that in both the main period, 1/59 to 12/78, and the subperiod, 1/65 to 12/78, the horizons of the same length of interest rates and inflationary expectations did not yield a unique match. It was suggested that the Fisher hypothesis applies only to equilibrium periods thus the study was not taken to be inconsistent with the long-run theory.

The main conclusion of the study has to be that if horizons are thought to be important, then the Fisher hypothesis stated with the duration of the interest rate matching the duration of expected inflation does not hold for the different durations along the yield curve.

In addition to using a multivariable expectation forecast, the results show an interest rate which includes additional variables as determinates provides a better fit and a smaller MSE than those equations without the additional variables. Finally, the real rate of interest is noted as not being constant across different horizons, but the explanation is left to question because the equation was not adequately controlled to discuss movements in the real rate.

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I was born July 20, 1948, to James Reed and Lois McCall Tipton in Knox County, Tennessee. I grew up and attended public schools in Chattanooga, Tennessee, graduating from Brainerd Senior High School in 1966.

I graduated from the University of Tennessee, Knoxville, with a Bachelor of Science in Economics in 1970. Then after spending three years as a regular Army officer, I served an internship with the Chicago Northwestern Transportation Company as a Treasury Analyst. Entering the University of Florida in September 1974, I graduated with a Master of Business Administration, concentrating in Finance in 1976. I married Barbara Ann Miller in June 1976. I then received a Master of Arts in Economics in 1978. I have accepted a joint appointment in both Finance and Economics from Baylor University, Waco, Texas beginning August 1980.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the Department of Economics in the College of Business Administration and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dean for Graduate Study and Research